

# The “Light Clocks” Thought Experiment and the “Fake” Lorentz Transformations

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**Abstract**—In this paper an alternative version of the well-known “light clocks” experiment is discussed. The so-called Lorentz transformations, backbone of the Special Relativity theory, are herein deduced by resorting to the above-mentioned experiment, albeit with a different meaning. Time dilation and length contraction are not considered as being real phenomena. Time, in fact, is peremptorily postulated as being absolute. Nonetheless, this strong assumption does not imply that instruments and devices of whatever kind, finalized to measure time, are not influenced by motion. In particular, although the “light clock” in the mobile frame ticks, so to say, more slowly than the one at rest, it can be easily shown how no time dilation actually occurs. The apparent length contraction is considered as being nothing but a banal consequence of a deceptive time measurement.

**Keywords**—Lorentz Transformations, Special Relativity, Light Clocks, Absoluteness of Time.

## I. INTRODUCTION

Firstly, it is fundamental to underline how time is considered as being absolute. Such an assumption, that could undoubtedly sound very anachronistic, does not imply that instruments and devices of whatever kind, finalized to measure time, are not influenced by motion and gravity [1] [2] [3]. Space is herein considered as being flat. The speed of light is considered as being constant and independent of the motion of the source.

Let’s consider two “light clocks”, initially at rest. At the beginning, the origins of the corresponding frames of reference, denoted by  $O$  and  $O'$ , are coincident. The homologous axes are parallel. We have two light sources, placed in  $O$  and  $O'$ , and two corresponding receivers, placed in  $R$  and  $R'$ , along the axes  $y$  and  $y'$  respectively. The distances between the sources and the corresponding receivers, identifiable with the heights of the clocks, are constant and equal to each other. Consequently,  $R$  and  $R'$  coincide when the frames are still at rest. When  $t=0$ , the clock whose frame is centered in  $O'$  starts moving rightwards, along  $x$  and  $x'$ , with a constant speed, denoted by  $v$ , whose value cannot equate that of light. The motion

consists in a simple translation. Simultaneously, both the sources are switched on: light is propagated along any direction, with a constant speed denoted by  $c$ . Let’s now suppose that when  $t=T$  a light signal is simultaneously received in  $R$  and  $R'$ . We contemplate two paradoxical scenarios.

The first scenario is qualitatively depicted in Figure 1.

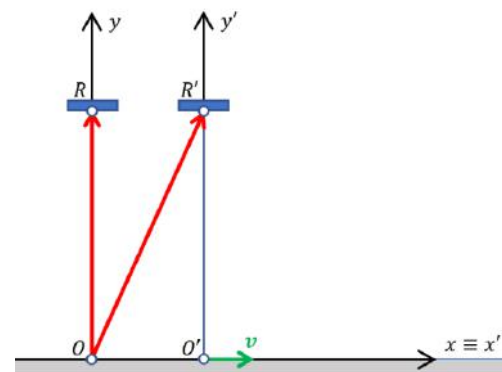


Figure 1. Variable Speed of Light

When  $t=T$ , denoting with  $l$  the height of the clocks, we can write the following:

$$\overline{OR} = \overline{O'R'} = l = cT \quad (1)$$

$$\overline{OO'} = vT \quad (2)$$

For the signal to be simultaneously received in  $R$  and  $R'$ , light should travel, along the linear path bordered by  $O$  and  $R'$ , with a greater speed whose value, denoted by  $c'$ , should be provided by the following relation:

$$c' = \frac{\overline{O'R'}}{T} = \frac{\sqrt{\overline{OR'^2} + \overline{OO'^2}}}{T} = \sqrt{c^2 + v^2} \quad (3)$$

Obviously, such a scenario would clearly contradict the hypothesis according to which the speed of light is constant. In this case, in fact, the speed of the light signal would depend on the motion of the source. Consequently, at least as far as the above-mentioned explanation is concerned, the signal cannot be simultaneously received.

The second scenario is qualitatively depicted in Figure 2.

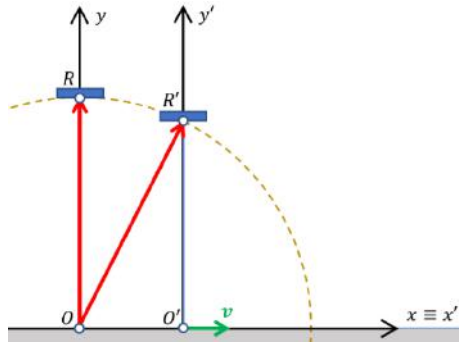


Figure 2. Variable "Light Clock" Height

This time, for the signal to be simultaneously received in  $R$  and  $R'$ , the mobile device should undergo a contraction along the direction orthogonal to the one along which the motion takes place.

If we denote with  $\gamma$  the so-called Lorentz factor [4] [5], the reduced height of the mobile "light clock" would be provided by the following relation:

$$\overline{O'R'} = \sqrt{\overline{OR'^2} - \overline{OO'^2}} = \overline{OR'} \sqrt{1 - \left(\frac{v}{c}\right)^2} = \frac{\overline{OR'}}{\gamma} \quad (4)$$

Very evidently, the scenario just imagined would clearly contradict the hypothesis according to which the heights of the clocks must remain constant. According to Special Relativity [6], in fact, the so-called "Lorentzian" contraction should exclusively occur along the direction of the motion. Consequently, once again, we cannot accept the possibility that the signal may be simultaneously received in  $R$  and  $R'$ . At this point, we are forced to admit that the signal must be received in  $R$  first, and then in  $R'$ .

Let's now investigate the real scenario.

On this purpose, we can suppose that when  $t=T'$  the signal is received in  $R'$ . The signal that followed the path bordered by  $O$  and  $R$  was previously received in  $R$  when  $t=T$ , with  $T < T'$ .

The real scenario is qualitatively depicted in Figure 3.

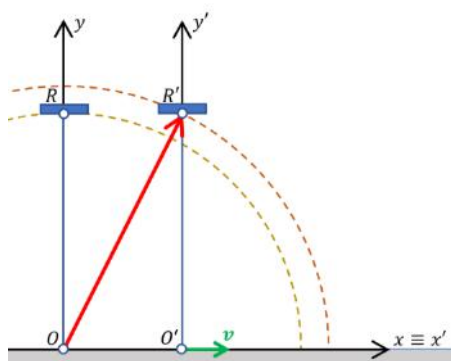


Figure 3. Real Scenario

Exploiting Figure 3, we can easily write the following:

$$\overline{OO'} = vT' \quad (5)$$

$$\overline{OR'} = cT' \quad (6)$$

$$cT = l = \sqrt{\overline{OR'^2} - \overline{OO'^2}} = cT' \sqrt{1 - \left(\frac{v}{c}\right)^2} \quad (7)$$

$$\frac{T'}{T} = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = \gamma \quad (8)$$

In theory, the phenomenon just analysed can recur indefinitely: suffice it to think that, for example, the receivers can consist in, and the sources can be replaced by, a couple of mirrors. Obviously, the measuring of the time elapsed clearly depends on the number of oscillations.

Consequently, although time keeps on being absolute, we can state that its measurement depends on the state of motion. In other terms, if we denote with  $t_f$  the measurement, that coincides with the absolute one, of the time elapsed in the frame at rest, and with  $t_m$  the measurement of the time elapsed in the mobile frame, we can write:

$$t_f = \gamma t_m \quad (9)$$

## II. THE "FAKE" TRANSFORMATIONS

Let's suppose that, at  $t=0$ , the mobile frame starts moving rightward with a constant speed equal to  $v$ .

Simultaneously, a light signal is sent from a generic point denoted by  $P$ .

The scenario is qualitatively depicted in Figure 4.

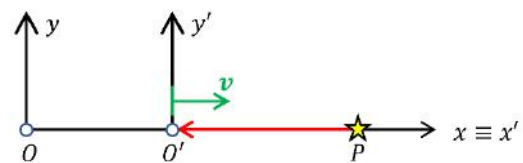


Figure 4. Direct Transformations

If we denote with  $t_{m,a}$  the time actually elapsed when the signal is received in  $O'$ , and with  $t_m$  the corresponding time measurement provided by the mobile light clock, we have:

$$\overline{OO'} = vt_{m,a} \quad (10)$$

$$t_m = \frac{t_{m,a}}{\gamma} \quad (11)$$

$$\overline{OO'} = \gamma vt_m \quad (12)$$

If we denote with  $x_{m,a}$  the absolute distance between  $O'$  and  $P$ , as soon as the signal is received in  $O'$ , and  $x_m$  the corresponding measurement deduced by exploiting the mobile clock, we have:

$$x_{m,a} = \overline{O'P} = ct_{m,a} \quad (13)$$

$$x_m = ct_m = \frac{ct_{m,a}}{\gamma} \quad (14)$$

$$\overline{O'P} = ct_{m,a} = \gamma x_m \quad (15)$$

If we denote with  $x_f$  the distance, absolute by definition, between  $P$  and  $O$ , we can write:

$$x_f = \overline{OP} = \overline{O'P} + \overline{OO'} = \gamma(x_m + vt_m) = \frac{x_m + vt_m}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \quad (16)$$

If we denote with  $t_f$  the time actually elapsed between the signal emission and the moment it is received in  $O$ , we can evidently write:

$$t_f = \frac{x_f}{c} \quad (17)$$

Moreover, very banally, from (14) we obtain:

$$t_m = \frac{x_m}{c} \quad (18)$$

From (16), (17), and (18), we have:

$$t_f = \frac{\frac{x_m}{c} + v \frac{t_m}{c}}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = \frac{t_m + \frac{vx_m}{c^2}}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \quad (19)$$

From (16) and (19), if we replace  $x_f$  with  $x$ ,  $x_m$  with  $x'$ ,  $t_f$  with  $t$ , and  $t_m$  with  $t'$ , we obtain the underlying well-known relations, that represent the so-called direct Lorentz Transformations [4] [5]:

$$x = \frac{x' + vt'}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \quad (20)$$

$$t = \frac{t' + \frac{vx'}{c^2}}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \quad (21)$$

Let's now suppose that at  $t=0$  the mobile frame starts moving leftward with a constant speed equal to  $v$ . Simultaneously, once again, a light signal is sent from a point  $P$ .

This time, very evidently, the signal reaches  $O$  first.

The scenario is qualitatively depicted in Figure 5.

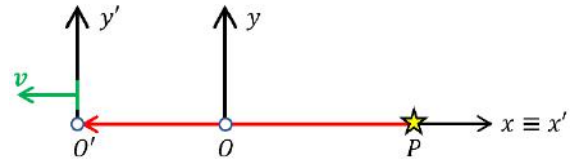


Figure 5. Inverse Transformations

At this point, maintaining the notation and exploiting the same line of reasoning we have followed in order to deduce the direct transformations, we can write:

$$x_f = \overline{OP} = \overline{O'P} - \overline{OO'} = \gamma(x_m - vt_m) = \frac{x_m - vt_m}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \quad (22)$$

$$t_f = \frac{\frac{x_m}{c} - v \frac{t_m}{c}}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = \frac{t_m - \frac{vx_m}{c^2}}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \quad (23)$$

In order to deduce the inverse transformations in their usual form, we have to replace, this time,  $x_m$  with  $x$ ,  $t_m$  with  $t$ ,  $x_f$  with  $x'$  and  $t_m$  with  $t'$ .

From (22) and (23) we obtain:

$$x' = \frac{x - vt}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \quad (24)$$

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \quad (25)$$

The replacements we have carried out in order to deduce the inverse transformations can be easily legitimised by means of a simple observation: as far as the scenario depicted in Figure 5 is concerned, the frame at rest receives the signal in advance with respect to the mobile one.

Consequently, it is as if the frame at rest were in motion towards the emission point.

It is worth underlining that, at this point, we can easily deduce a shorter and more elegant version of the transformations by exploiting the hyperbolic functions.

Let's carry out the following position:

$$\frac{v}{c} = \tanh \varphi \quad (26)$$

From the previous identity we can banally deduce the so-called boost parameter:

$$\varphi = \tanh^{-1} \left( \frac{v}{c} \right) \quad (27)$$

From (26) we immediately obtain:

$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = \frac{1}{\sqrt{1 - \tanh^2 \varphi}} = \cosh \varphi \quad (28)$$

At this point, for example, the inverse transformations can be rewritten as follows:

$$x' = x \cosh \varphi - ct \frac{v}{c} \cosh \varphi = x \cosh \varphi - ct \sinh \varphi \quad (29)$$

$$ct' = ct \cosh \varphi - x \frac{v}{c} \cosh \varphi = -x \sinh \varphi + ct \cosh \varphi \quad (30)$$

Finally, from the previous two relations we can immediately obtain the well-known underlying form:

$$\begin{pmatrix} x' \\ ct' \end{pmatrix} = \begin{pmatrix} \cosh \varphi & -\sinh \varphi \\ -\sinh \varphi & \cosh \varphi \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix} \quad (31)$$

### III. BRIEF CONCLUSIONS

The Lorentz Transformations are herein deduced by exploiting the well-known “light clocks” thought experiment. As provocatively suggested by the title, we highlight some contradictions that arise from hypothesizing that the above-mentioned devices, coherently with the well-known hypotheses under which the experiment is commonly carried out, could receive the light signals simultaneously. It is worth underlining how, in the light of some noteworthy criticisms concerning Special Relativity [7] [8], we have elsewhere carried out a further alternative deduction of the Lorentz Transformations [9], by hypothesizing a closed Universe, belonging to the so called oscillatory class [10] [11], globally flat, characterized by four spatial dimensions [12].

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